

ABSTRACT

The main purposes of a Heating, Experimental values for Pr (0.71) have been obtained from the mean velocity and temperature profile data of Simpson [5] and Whitten [6] for various condition like blown, unblown, and sucked turbulent incompressible air in above mentioned boundary condition. The value of Pr no. lies in the range of $1 > Pr > 1$, means for inner similarity region, $Pr < 1$ while $Pr > 1$ in the outer similarity region. These results are in agreement with Ludwig's [2] pipe results and show no effect of blowing or suction on Pr . The Jenkins model [21] is found to describe the variation of Pr , which accounts for the unequal loss of momentum and thermal energy from an eddy in Sight for $Pr = 1$ fluids in the inner region within experimental uncertainty of the data. Also using Hinze's suggestion that the diffusion of heat might be a combination of gradient and large eddy transport, a new model is developed to account for $Pr < 1$ in the outer region. Predictions based on these models lie within the uncertainty band of the experimental results and indicate no effect of blowing or sucking on Pr .

Keywords: Pr No., Turbulent flow, Suction and injection.

I. INTRODUCTION

The primary requirement of As IS well known, there exists at the present time no purely theoretical solution of the fluid dynamics of the turbulent boundary layer. Consequently there is no theoretical solution available for heat transfer in the turbulent boundary layer. In the momentum problem the "eddy viscosity" remains unknown while the "eddy conductivity" is unspecified in the case of heat transfer. The classical approach to obtaining the transport mechanism for the heat transfer problem follows the laminar approach; namely, the momentum and thermal transport mechanisms are related by a factor, the Prandtl number Pr . Hence, combining the laminar and "eddy" viscosities one obtains the Boussinesq relation

$$\frac{\tau g_c}{\rho} = (\nu + \epsilon_M) \frac{\partial U}{\partial y}$$

for the shear stress and the analogous relation

$$\frac{\dot{q}''}{\rho c_p} = - \left(\frac{\nu}{Pr} + \frac{\epsilon_M}{Pr_t} \right) \frac{\partial T}{\partial y}$$

for the heat flux. The quantity Pr , is known as the turbulent Prandtl number.

Thus if one knows the eddy viscosity and the turbulent Prandtl number the heat transfer problem can be solved. A number of experimental and theoretical investigations have been devoted to obtaining the eddy viscosity. Only a few studies have been made of the turbulent Prandtl number. No previous experimental studies have been reported on the effect of blowing or suction on Pr

Review of previous works

Kestin and Richardson [1] recently reviewed the status of the turbulent Prandtl number. They found that the results from the few experimental studies were in conflict. The results from mercury experiments in pipes indicated that $Pr_t > 1$ while gas experiments in pipes showed $Pr_t < 1$. Thus it is not clear whether the turbulent Prandtl number is completely independent of the molecular Prandtl number. The results of Ludwig [2], as shown in Fig. 1, and others [1] for air flowing in a pipe do not agree.

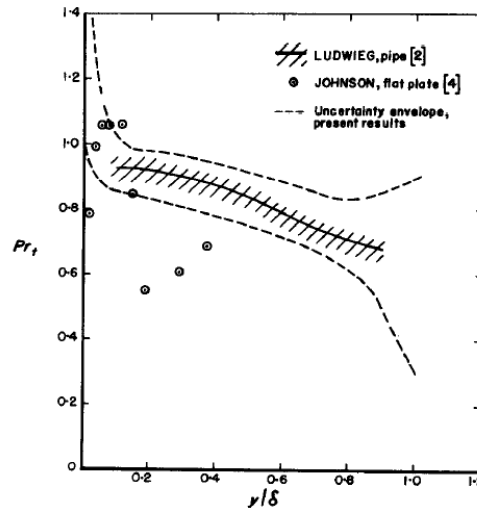


FIG. 1. Comparison of experimental results.

In a brief account of these investigations, Kestin and Richardson [1] concluded that Ludwig's results are the most reliable for air flowing in a pipe. The flow at the center of a pipe does not include regions of intermittent wake-like flow, such as occur in the outer region of an external boundary layer. On extrapolation of Ludwig's results on the basis of the reciprocal of distance from the wall, they found that his measured values were asymptotic to a turbulent Prandtl number of 0.5 at large distances from the wall. This is in agreement with the value of 0.5 deduced by Fage and Faulkner [3] from the wake of a cylinder and by Reichardt [1] in a free jet. The value of 0.5 is also obtained from Taylor's vorticity transport theory [3], which gives further support to the trend of Ludwig's results. The only experimental study of Pr_t on a flat plate with a constant free-stream velocity known to the authors was reported by Johnson [4], who used hot-wire anemometers to determine the distribution of velocity and temperature fluctuation levels. He studied the temperature distribution downstream of an unheated starting length where the thermal boundary layer was contained at all times in an inner fraction of the momentum boundary layer, providing no information about the outer region. Johnson compared the turbulent shearing stress and the heat flux obtained by hot wire measurements with those generated from mean velocity and temperature distributions, finding a 50 per cent discrepancy in the shearing stresses and good agreement for the heat fluxes. He noted that the skin-friction coefficients obtained by several independent methods did not agree. The anomalous behaviour was attributed to three-dimensionality of the flow. As shown on Fig 1, the scatter of the Pr_t data points is considerable. Even so, the average of these results near $y/S \times 0-1$ is in fair agreement with Ludwig's results. As concluded by Kestin and Richardson, the question of the turbulent Prandtl number is unresolved and merits further experimental investigation not only for air but for fluids of a wide range of molecular Prandtl number.

II. OBJECTIVES OF THE PRESENT WORK

There is little consistent experimental evidence as to the distribution of Pr_t in the boundary layer on a flat plate for air. There exists no published experimental study of the effect of blowing and suction on the turbulent Prandtl number. The Pr_t can be determined by measurements of velocity and temperature distributions in the boundary layer, and the heat flux and shear stress at the wall. Such measurements have been reported by Simpson [5] and Whitten [6] for a wide range of blowing and suction conditions with constant free-stream velocity and constant wall temperature. The blowing conditions were such as to hold the blowing parameter B constant. The experimental Stanton number and skin-friction coefficient results associated with these data have been previously

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described [5-81]. A description has been given of the flow characteristics associated with these data. In broad terms, the objectives of the present work are to determine the turbulent Prandtl number Pr_t for air from the data of Simpson and Whitten and to compare these results with available theories.

III. CONCEPT

Reynolds [1] was the first to assume that $Pr_t = 1$ on the basis of a heuristic argument which notes that in a fully turbulent field, both momentum and heat are transferred as a result of eddies. From Figs. 6-8 one can see that Reynolds' argument fails to hold in detail throughout the boundary layer. The local value of $Pr_t > 1$ near the wall ($y^+ < 150$) where the small scale turbulence is strongly affected by molecular kinematic viscosity. The $Pr_t < 1$ in the outer region ($y^+ > 0.05$) where v has little influence. Likewise, it is suspected that A affects the transport of heat near the wall and has little influence in the outer region.

6.1 Inner region, $Pr_t > 1$: background information

Jenkins [21] devised a model to account for the unequal loss of momentum and thermal energy from an eddy in flight between mixing points for a $Pr_t > 1$ fluid. For coherence the main points of this model are presented. He argued that if the temperature of the eddy did not change in flight, then the definition of the mixing length. Treating the effects of molecular viscosity on an eddy in flight in the same manner as the effects of molecular thermal conductivity, he obtained the following relation with experiment near the wall and fails in the outer region. This model agrees with the idea that small scale wall turbulence is governed by molecular properties (near wall) but fails to account for the large eddy motion in the outer region. The following hypothesis accounts for the effect of this large eddy structure. As pointed out by Hinze [14] from the work of Townsend [20], the transfer of mainstream momentum, a vector quantity, appears to be a velocity gradient related process associated with small scale turbulence. On the other hand, turbulence energy, a scalar quantity, appears to be mostly diffused by the large eddies [S, 203, at least in the outer part of the boundary layer where the diffusion term in the turbulence energy equation is most important. This part of the turbulence energy diffusion has been represented. The effective velocity at which the turbulence energy is transported in the y -direction. It is not entirely surprising that the Jenkins model agrees, within experimental uncertainty, large eddies [22]. To determine the value and variation of the quantity q through a boundary layer, the following approximate model is proposed. Bradshaw [18] has noted that at the outer edge of a self-similar boundary layer flow, such as the flow considered here, VP is equal to the mean rate of propagation of turbulent fluid into the freestream—the “entrainment velocity”. Although no information is available for the effect of blowing or suction on $G(q)$ and z/p^+ , it is assumed that this resulting VP/U , variation, which is roughly linear in q , applies for all cases considered here. It is assumed that $1 + t$, the mixing length l , and the mean temperature gradient aT/ay are related. One is now in a position to calculate Pr_t , and hence CH from the turbulent flow structure of the boundary layer and the molecular Pr . In the inner region Pr_t is found to be described within experimental uncertainty, by equation (26) while equation (35) describes the outer region. as suggested by Rotta [12] for unblown flows. Using equation (38) the velocity “law of the wall” for the inner region, and the “velocity defect law” for the outer region, he calculated the Reynolds analogy factor $St/(C_f/2) = 1.16$ for $Pr_t = 0.72$. Whitten [6] obtained $St/(C_f/2) = 1.16$ from the experimental heat transfer and skin friction results associated with the present profiles. Equation (38) is seen to agree with the calculated results within 0.05, to be within the uncertainty envelope of the experimental results, and to produce a Reynolds analogy factor agreeing with the experimental value. Hence equation (38) should be a reliable Pr_t distribution for $0.1 < q < 1.0$ for all blown, sucked and unblown constant freestream velocity flows.

IV. CONCLUSION

The designed experimental turbulent Prandtl number results from the velocity profiles of Simpson [5] and temperature profiles of Whitten [6] have been presented for constant free-stream velocity constant B flows ($-0.48 < B < 6.78$). A description has been given of the procedure used in obtaining these results. Near the wall in the region of U^+ vs. y^+ similarity, the molecular viscosity and Prandtl number and the small scale turbulence govern the momentum and heat transport. $Pr_t > 1$ and correlates best with the inner variables $E&J$ and y^+ . In the outer region $Pr_t < 1$ and is correlated against the characteristic coordinate ~ 1 . No effect of blowing or suction on Pr_t can be seen from present experimental results.

V. REFERENCES

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